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No. 2017/7

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*Helena Cruz Castanheira, François Pelletier and Igor Ribeiro*



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## PREFACE

This technical paper presents a sensitivity analysis of some assumptions used in the Bayesian framework for projecting life expectancy at birth. The paper also showcases the changes that were made while producing the official estimates and projections of the *2017 Revision* of the *World Population Prospects*. The results of any Bayesian estimation procedure are a combination of information known *a priori* about the topic being analysed with information from the data, resulting in posterior distributions. The *a priori* information is represented by prior distributions pertaining to individual parameters of the model. The Bayesian hierarchical model for projecting life expectancy at birth was used in the *2012, 2015* and *2017 Revisions*. In the *2015 Revision*, prior distributions were adjusted for close to 30 per cent of the countries for which this model was used because the unadjusted posterior estimates of life expectancy at birth were deemed too high or too low compared to countries with similar levels of life expectancy. Furthermore, through an analysis of the uncertainty bounds, it was observed that the bounds were highly asymmetrical in several countries and that the priors needed to be revised. For a better understanding of these issues and to guide the choice of assumptions used for the projection of life expectancy in the *2017 Revision,* we performed an analysis of the sensitivity of the resulting projections to changes in the parameters of the prior distributions of the Bayesian estimation model. Using insights gained from the sensitivity analysis, we updated the specification of the prior distributions. The analysis also led to the inclusion of historical data for periods prior to 1950 in estimating the Bayesian model of the gap in life expectancy at birth. In general, the projections for countries with relatively low levels of life expectancy were more sensitive to changes in the specification of the prior distributions. The changes implemented in this revision have improved the model fit and the consistency of the projected trends, resulting in a decrease in the number of countries deemed to require *ad hoc* adjustments in the *2017 Revision*. This paper illustrates the extent to which both the median trajectory and the uncertainty bounds are sensitive to the choices made for the prior distributions of specific parameters.

The authors gratefully acknowledge Patrick Gerland, Hana Ševčíková and Mark Wheldon for their technical support and helpful comments during the analytical phase. The authors also acknowledge the insightful inputs of colleagues in the Population Estimates and Projection Section: Lina Bassarsky, Danan Gu, Neena Koshy, Cheryl Sawyer, Thomas Spoorenberg and Guangyu Zhang.

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# A SENSITIVITY ANALYSIS OF THE BAYESIAN FRAMEWORK FOR THE PROJECTIONS OF LIFE EXPECTANCY AT BIRTH

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## 1. Introduction

A Bayesian hierarchical model of temporal changes in life expectancy at birth was used by the Population Division of the United Nations for projecting future value of life expectancy as part of the *2012, 2015 and 2017 Revisions* of the *World Population Prospects* (WPP) (United Nations, 2013, 2015a, 2017a). Every two years, the United Nations updates its estimates and projections of mortality rates and life expectancy. The latest revision of the *World Population Prospects* was released in June 2017 (United Nations, 2017a). To gain a better understanding of the Bayesian model and to inform the choice of assumptions concerning prior distributions of model parameters affecting the projections of life expectancy at birth in the *2017 Revision*, we analysed the sensitivity of projected trends derived from the estimated model to changes in the assumptions used for fitting the model.

This exercise has demonstrated the utility of analysing the sensitivity of model-based estimates and projections to changes in the assumptions used in estimating the underlying statistical model. The Bayesian framework incorporates *a priori* information (or assumptions) about the process being studied with information gleaned from the data to produce posterior distributions. By assumption, the parameters of the model are not fixed constants but rather represent random draws from a probability distribution. Thus, it is necessary to specify prior distributions for every parameter based on available information about the phenomena being analysed. In this paper, we perform a sensitivity analysis concerning parameters of the prior distributions of the Bayesian hierarchical model used for projecting female life expectancy at birth. In addition, a critical choice regarding the historical data used for estimating the model of the sex gap in life expectancy is assessed.

The objective of the sensitivity analysis was to address certain issues encountered in projecting life expectancy for the *2015 Revision*. First, the life expectancy projections for close to 30 per cent of the countries or areas were deemed too high or too low, requiring *ad hoc* adjustments to obtain more plausible results (United Nations, 2015a). Second, the prediction intervals (PIs) were highly asymmetrical around the median trajectories for several countries, with the mean value diverging significantly from the median in some cases. Third, the levels and trends of several countries had outcomes that seemed unlikely, especially for the upper 95-per cent prediction intervals. Upper bounds for countries such as Guinea, India, Mali or Senegal, for instance, were similar or exceeded the upper bounds for countries such as Australia, Japan, Switzerland or the Republic of Korea by 2100. Lastly, a set of influential outliers among the available data points, including in some cases negative gains in life expectancy, had strongly affected the parameter estimates for some countries.

In this paper, we describe a sensitivity analysis that was used to inform the specification of the prior distributions used for the Bayesian hierarchical model of the *2017 Revision*. The model and its parameters are briefly summarized and the sensitivity analysis is explained.

## 2. Methodological Approach

### *2.1. The UN Probabilistic Model of Projecting Life Expectancy*

The method used in the *2017 Revision* for projecting life expectancy at birth and the corresponding age-specific mortality rates consist of three separate steps. First, the life expectancy at birth of females from 2015-2020 until 2095-2100 is projected using a Bayesian hierarchical model (Raftery et al., 2013). Second, the life expectancy of males is obtained by modelling the difference between female and male life expectancy in 1950-2015 using a linear regression (Raftery, Lalic and Gerland, 2014), and then projecting the life expectancy of males based on the parameters resulting from the regression and the projected female life expectancy. Finally, the age specific mortality rates are projected based on the given levels of life expectancy using either the modified Lee-Carter method (Li, Lee, and Gerland, 2013), the pattern of mortality decline method (Andreev, Gu and Gerland, 2013) or model life tables depending on the quality and availability of the mortality data by age and sex in 1950-2015 (United Nations, 2017b; Gu, Pelletier and Sawyer, 2017).

The Bayesian hierarchical model was estimated following the methodology introduced by Raftery et al. (2013). In this context, the double-logistic function previously used by the United Nations for deterministic projections (United Nations, 2000, 2006) was incorporated to the Bayesian framework. The double-logistic function models five-year gains in life expectancy assuming that countries with the lowest and highest life expectancies tend to improve life expectancy more slowly than countries in the middle. This model follows the assumption of logistic growth observed in many biological and social systems, especially when an innovation is adopted, and it is able to capture the secular trends in life expectancy experienced due to improvements in health behaviour and technology, nutrition, and sanitation across countries (Marchetti, 1997). In summary, the model assumes that the gains in life expectancy increase as life expectancy increases until it reaches a saturation point, then decrease until it reaches an asymptotic rate of increase, defined as $z^c$. Formally, as shown in Raftery et al. (2013), this process is summarized with the equation:

$$g\left(e_{c,t} \mid \theta^c\right) = \frac{k^c}{1+\exp\left(-\frac{A_1}{\Delta_2^c}\left(e_{c,t}-\Delta_1^c - A_2\Delta_2^c\right)\right)} + \frac{z^c - k^c}{1+\exp\left(-\frac{A_1}{\Delta_4^c} * \left(e_{c,t}-\left(\Delta_1^c+\Delta_2^c+\Delta_3^c\right)-A_2\Delta_4^c\right)\right)} \qquad \text{(Eq.1)}$$

Life expectancy is projected stochastically by a random walk with drift, with the drift being $\left(e_{c,t} \mid \theta^c\right)$ , as defined in:

$$e_{c,t+1} = e_{c,t} + g\left(e_{c,t} \mid \theta^c\right) + \varepsilon_{c,t+1} \qquad \text{(Eq. 2)}$$

In the equations above, $A_1$= ln (81) and $A_2$ = 0.5 as defined in Raftery et al. (2013). The parameter $k^c$ is the level at which the first growth process saturates, $\Delta_1^c$ is the life expectancy at 10 per cent of the first growth process, $\Delta_2^c$ is the number of years between the life expectancy at 10 and 90 per cent of the first growth process, $\Delta_3^c$ is the number of years between the 90 per cent saturation point of the first process and the 10 per cent of the second saturation point, $\Delta_4^c$ is the difference in life expectancy years between the life expectancy at 10 per cent and 90 per cent of the second saturation point ($z^c - k^c$), and $z^c$ is the asymptotic average rate of increase in female life expectancy per five-year period. Since $k^c$ is by definition greater than $z^c$, the numerator of the second equation is always negative resulting in the bell shape form of the two logistics. The error $\varepsilon_{c,t+1}$ is the random perturbation of the projection, which makes $e_{c,t+1}$ stochastic resulting in a random walk with a drift, given by $g\left(e_{c,t} \mid \theta^c\right)$. This double-logistic model is estimated using a Bayesian hierarchical approach (Raftery et al., 2013), with the countries nested in the world, so the world's experience is used to inform the country's parameters. The Bayesian model requires specifying prior distributions of the world mean and variance for every parameter, for which the sensitivity analyses were performed (see section 2.2). The posterior distributions of the parameters were approximated by Markov Chain Monte Carlo (MCMC) (see Raftery et al. 2013 for more details), with ten chains and 160,000 iterations in each chain.

### *2.2. Simulations*

A total of four simulations are presented in this technical paper. The main objective is to compare the results of each simulation with the final model used in the *2017 Revision* and observe to what extent the differences between the simulation and the final model affect the result of the projections. In the first simulation, the five UN models of mortality decline (very fast, fast, medium, low and very low) used for deterministic projections in the past (United Nations, 2000, 2006) are updated and used for defining the mean and standard deviation of the world's prior distributions. This update consisted in incorporating the most recent estimates of female life expectancy at birth (from 2000 to 2015) and using the latest updated estimates for the period 1950-2000 as input data to refit the five previous UN models. When estimating the models, the highest gain of each country was obtained, after eliminating outliers (mean plus or minus three standard deviations) and negative gains, and the 90, 75, 25, and 10 percentiles were estimated based on the world distribution of maximum gains. In this regard, the double logistic curves were estimated based on the data of the countries that had their maximum gains greater than or equal to the 90th and 75th percentiles for the very fast and fast models, and lower than or equal to the 25th and 10th percentiles for the low and very low models. The medium model was predicted in the sample with all countries. The double logistic functions were estimated using the formula in Raftery et al. (2013), equation 1 above, and the sum of the squared errors were minimized using the BB package optimizer in R (Varadhan and Gilbert, 2009).

The second simulation consists of changing the upper limit of the maximum sum of deltas, in which MCMC proposals outside the specified limit are rejected. In this regard, the estimates with the upper limit of 110 years, used in the *2015 Revision*, are compared to 86 years, used in the *2017 Revision*. Restricting the maximum sum of deltas to 86 years, means that the mortality transition of countries must be completed at the female life expectancy level of 86 years, after which the increments in life expectancy are mainly driven by the asymptotic average rate of increase ($z^c$).

A third simulation was prepared in order to observe the impact of changing the upper limit of the prior of the asymptotic average rate of increase, the *z* factor in equation 1. This upper limit was not changed in the *2017 Revision* and is the same across the *2012*, *2015 and 2017 Revisions*, but it was considered important to show the extent to which the upper bound of the *z* factor affects the results of the projection of female life expectancy and the countries that are more affected by those changes. The simulation changes the upper bound from the 1.3 female life expectancy years per decade, used in the *2012*, *2015 and 2017 Revisions*, to 2.3 as proposed in Oeppen and Vaupel (2002) and in the original Raftery et al. (2013) paper. On a five-year basis, the values were set at 0.65 and 1.15, respectively (see table 1 and figure 3 in section 3 below).

The fourth simulation reviews the specifications of outliers of the database used to estimate the models. In the *2017 Revision*, the initial database consisted of 2,502 five-year gains in female life expectancy ($g(e_{c,t}) = e_{c,t+1} - e_{c,t}$) for 182 countries[1] from 1950-1955 to 2010-2015, and supplemental historical data for 29 countries that had reliable life expectancy estimates before 1950 (for more information, see United Nations, 2017b). The data set contained a total of 111 country-period negative gains, caused by mortality crises and exceptional mortality conditions. These negative gains influenced the predicted parameters of countries, especially the ones with no supplemental historical data, and, consequently, were influencing their trajectories in the future. Given that these negative gains were mostly observed for countries that had already reached or surpassed the life expectancy experienced in the period before the loss[2], the database was truncated for observations equal to or greater than zero so that the loss in life expectancy would not influence the mortality trajectories of countries in the future. In addition, countries with life expectancy gains greater than three standard deviations from the mean, which are gains greater than seven years (a total of 20 country-period observations), were also considered outliers and removed from the data. The final number of country-period observations from which the Bayesian Hierarchical model was estimated is 2,371.

### *2.3. Changes in the estimation of the female-male gap in life expectancy at birth*

In the *2015 Revision* (United Nations, 2015a), it was deemed necessary to increase the threshold of the female life expectancy to 86 years, that is the threshold at which the female-male gap remains constant, compared to 83 in the original model developed by Raftery et al. (2014). This change permitted slightly more convergence in the sex differential of mortality for some countries with higher levels of life expectancy. In addition, different levels of the minimum and maximum bounds of the gap were tested; in the previous revision, the lower bound had been set at -1 and in the *2017 Revision* it was set to zero. In the present study, the threshold of 86 years is maintained and the historical data for periods prior to 1950 for a number of countries were added to the dataset for estimation of the coefficients of the life expectancy gap model. The current sensitivity analysis only refers to the incorporation of the historical data.

[1] Out of a total of 233 countries or areas in the world, the database consisted of 182 countries, excluding 32 countries with less than 90,000 inhabitants, Syria, currently in a mortality crisis, and 18 countries with a maximum HIV prevalence of 5% or above between 1980 and 2015 among persons aged 15 to 49 years (for further details, see United Nations, 2017b).

[2] The negative gains that were observed in the data with the 182 countries refer to a total of 66 countries. Among these, only Belize, Côte d'Ivoire, and Iraq had not achieved in 2010-2015 the life expectancy levels prevailing before the loss.

## 3. Results

### ***3.1. Assessing the goodness of fit across simulations***

Table 1 shows the goodness of fit for the different simulations. It presents the extent in which the total coverage (a measure showing the extent in which the empirical data are within the 80, 90 or 95 per cent bounds), root mean square error or mean absolute error are changed when the previous definitions of the prior distributions are used instead of those from the *2017 Revision*. The simulations were done by changing specific parameters of the final model of the *2017 Revision*. The final settings of the *2017 Revision* (WPP 2017 in table 1) has the updated mean and standard deviation of the normal prior distributions for the world parameters, the upper limit of the sum of deltas set to 86 years, the upper limit of the *z* factor set to 0.65, and the data set limited to five-year life expectancy gains between zero and seven. Simulation 1, for instance, has all the settings of the *2017 Revision*, except the mean and standard deviation of the normal prior distributions for the world parameters of the *2015 Revision*.

Table 1. Assessing the goodness of fit of simulations: total coverage, root mean square error (RMSE) and mean absolute error (MAE)

| *Simulations* | *Total Coverage* 80% | 90% | 95% | *Root Mean Square Error* | *Mean Absolute Error* |
|---|---|---|---|---|---|
| Simulation 1: WPP 2015 mean and standard deviation of the normal priors for world parameters | 0.87 | 0.93 | 0.96 | 0.71 | 0.50 |
| Simulation 2: WPP 2015 sum of deltas upper limit restricted to 110 years | 0.87 | 0.93 | 0.96 | 0.71 | 0.50 |
| Simulation 3: *z* factor upper limit restricted to 1.15 | 0.87 | 0.94 | 0.97 | 0.70 | 0.49 |
| Simulation 4: WPP 2015 outlier restriction (-5,10) | 0.89 | 0.93 | 0.96 | 0.99 | 0.64 |
| WPP 2015 | 0.90 | 0.93 | 0.95 | 0.97 | 0.61 |
| WPP 2017 | 0.87 | 0.93 | 0.96 | 0.72 | 0.51 |

*Data Source:* United Nations, Department of Economic and Social Affairs, Population Division (2017c), *World Population Prospects: The 2017 Revision.*

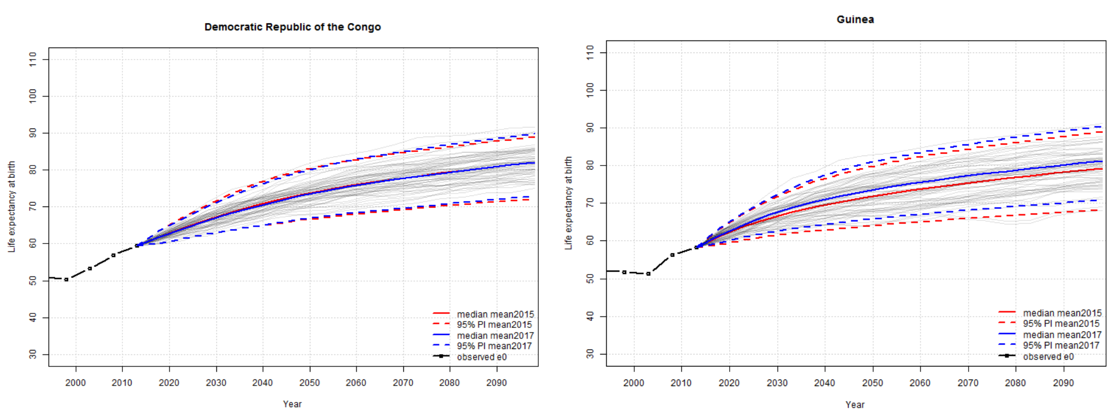
Overall, it can be observed that the changes in the prior distributions and data specifications do not affect the total coverage within the 80, 90 or 95 per cent bounds. The largest changes were for the outliers' definition of the *2015 Revision* (simulation 4), from 0.87 to 0.89, for the 80 per cent bound; however, for the 90 and 95 per cent coverage the values with the old and new specifications were the same being 0.93 and 0.96 respectively. The coverage of the priors used in the *2015 Revision* are very similar to those used in the *2017 Revision*, being 0.90, 0.93 and 0.95 for the 80, 90 and 95 per cent coverage for the *2015 Revision* and 0.87, 0.93 and 0.96 for the *2017 Revision*.

Table 1 also shows the root mean square error (RMSE) and the mean absolute error (MAE), which provide the goodness of fit related to the dispersion of the errors. The lower the magnitude of the RMSE and MAE, the better the fit of the model. Both measures were reduced considerably in the *2017 Revision* (WPP 2017) compared to the *2015 Revision* (WPP 2015), mainly driven by the changes in the specification of the outliers (simulation 4). For the other simulations, however, the changes in the RMSE and MAE are of small magnitude. In this regard, the visualization of the results as presented in the next section and its comparison across countries and within regions are important for verifying the feasibility of the priors and changes implemented in the *2017 Revision.*

### *3.2. Country-specific Results*

Figures 1 to 4 show the effects of the four simulations for selected countries. The effect of simulation 1 is presented in figure 1 for the Democratic Republic of the Congo (DRC Congo) and Guinea. It can be observed that, when updating the mean and standard deviation of the world mean prior distributions, the median trajectory of DRC Congo did not change, but the median trajectory of Guinea increased by about three years from 2030-2035 onwards. Overall, this change did not affect the projected trajectories in many countries. However, trajectories of life expectancy at birth were slightly shifted upwardly in a few countries where mortality levels are currently higher.

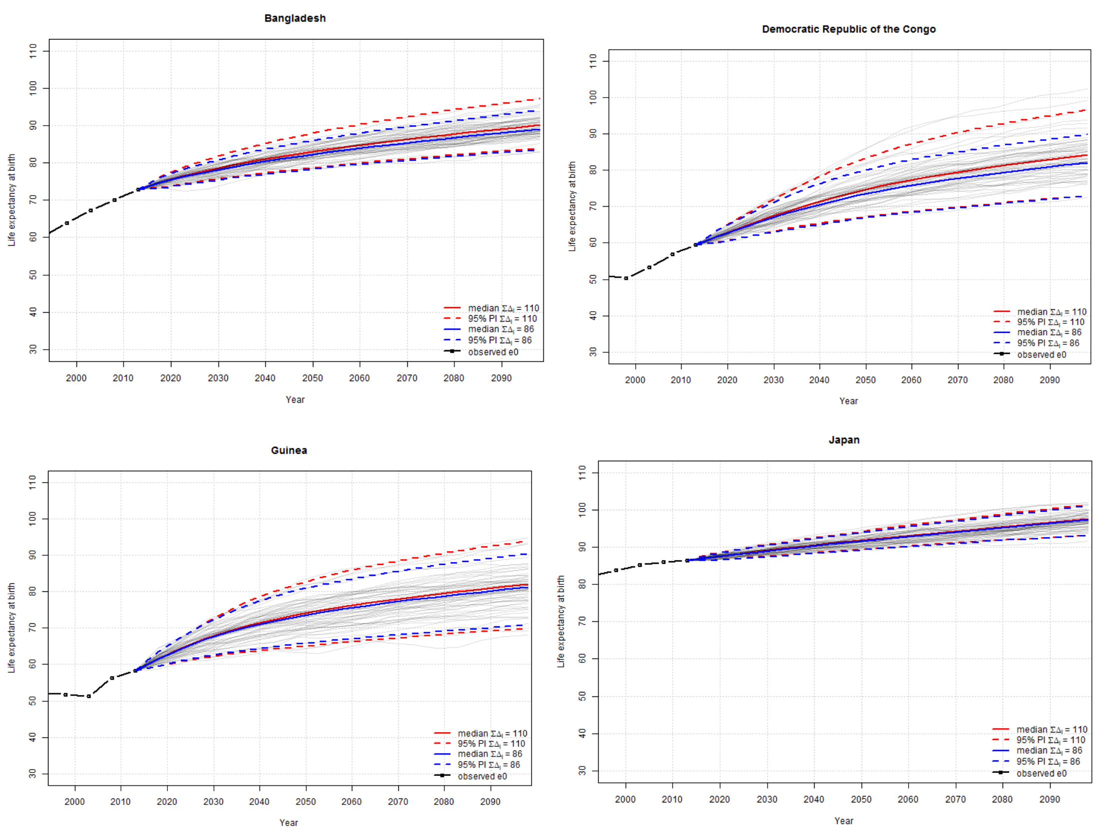
**Figure 1. Life expectancy at birth (in years), simulation 1 and *2017 Revision*, Democratic Republic of the Congo and Guinea, 1995 to 2100**




*Data Source:* United Nations, Department of Economic and Social Affairs, Population Division (2017c). World Population Prospects: The 2017 Revision.

Decreasing the upper limit of the sum of deltas (simulation 2) affected mainly countries with lower levels of life expectancy at birth, which are countries in early phases of the mortality transition. As illustrated in figure 2 for Bangladesh, DRC Congo and Guinea, reducing the upper bound of the sum of the deltas from 110 years to 86 years had a considerable influence on the upper bound of the 95 per cent prediction intervals for countries with relatively low levels of life expectancy. The change also affected their median trajectories to different extents. However, the change in the sum of deltas had hardly any effect on Japan, a country that is more advanced in the mortality transition.

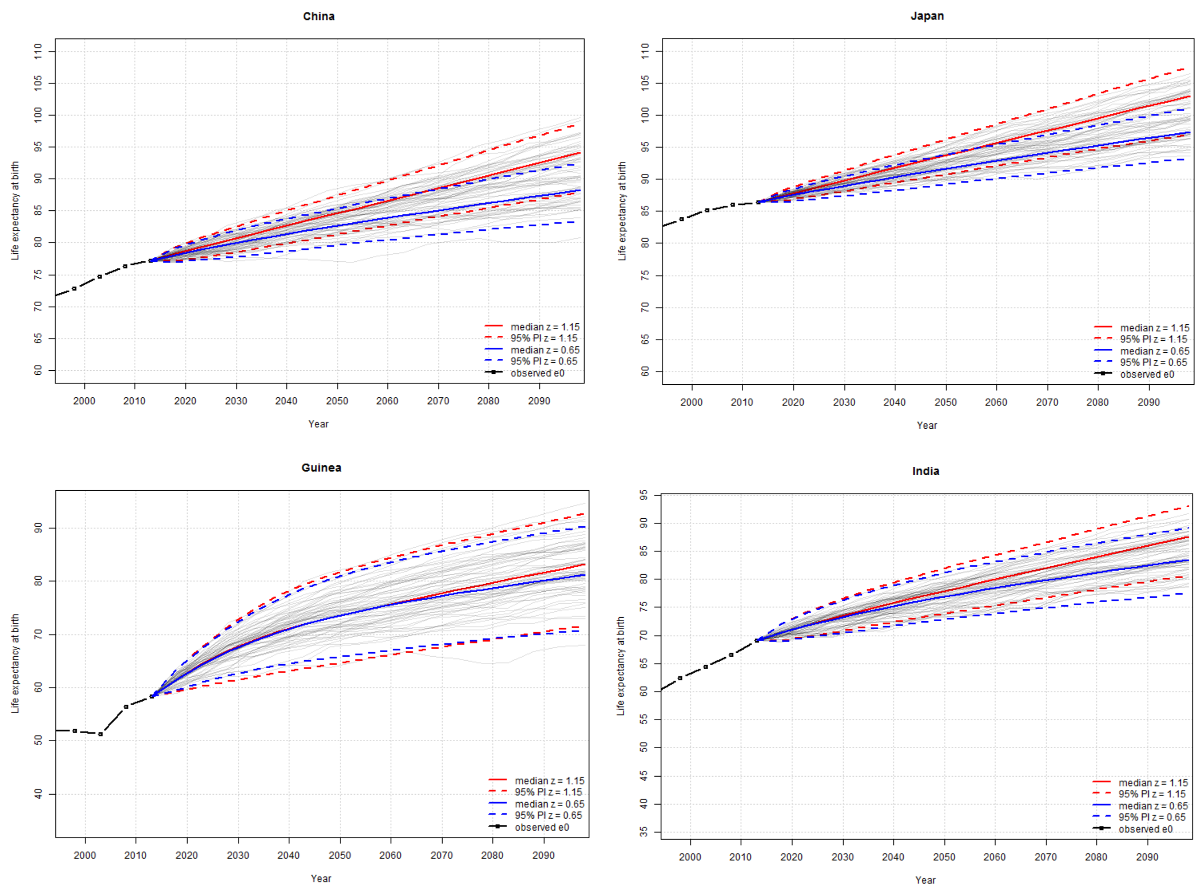
**Figure 2. Life expectancy at birth (in years), simulation 2 and 2017 Revision, Bangladesh, the Democratic Republic of the Congo, Guinea and Japan, 1995 to 2100**




*Data Source:* United Nations, Department of Economic and Social Affairs, Population Division (2017c), *World Population Prospects: The 2017 Revision.*

Figure 3 shows the effect of setting different upper limits of the *z* factor for China, Japan, Guinea and India (simulation 3). It can be observed that the change from 0.65 to 1.15 years per five-year period produced significant shifts in the median trajectory and associated bounds of both China and Japan from 2030 onwards; the variations in India are slightly smaller and started in later periods, while in Guinea the changes are minimal. Overall, the modifications in the $z$ factor affected many countries but to a lesser extent those with relatively low levels of life expectancy at birth.

**Figure 3. Life expectancy at birth (in years), simulation 3 and *2017 Revision*, China, Japan, Guinea and India, 1995 to 2100**




*Data Source:* United Nations, Department of Economic and Social Affairs, Population Division (2017c), *World Population Prospects: The 2017 Revision.*

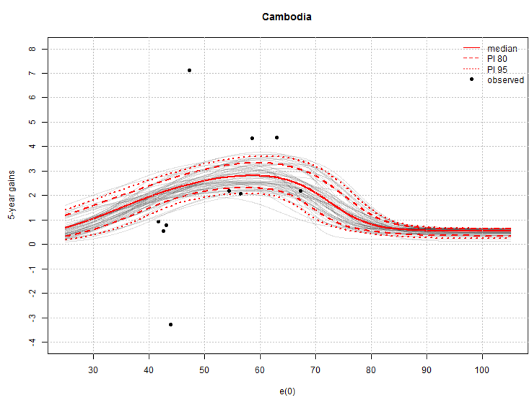
The sensitivity of estimates to negative gains and outliers can be illustrated by comparing Cambodia and Switzerland (simulation 4). Figure 4 shows the difference in the estimation of the median double logistic curves of the two countries using the 2015[3] and 2017 restrictions for outliers, and the most recent 2017 data. It can be observed that the values of the median parameters of both countries have changed, however the change in the double logistic curve of Cambodia is more expressive. This difference is reflected in the estimated median female life expectancy of the countries in 2100. In Cambodia, it increased 1.4 years in the *2017 Revision*

[3] In the *2015 Revision*, outliers were considered the country-periods with five-year gains lower than -5 or greater than 10, a total of four observations in the 2017 data.

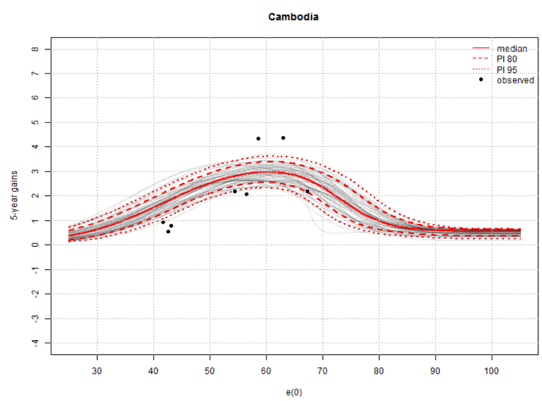
when the influential outliers were removed, from 86.5 to 87.9, and in Switzerland it increased half a year, going from 94.8 to 95.3. The difference in the impact of the outliers' restriction in the two countries is likely to be driven by the difference in the countries' level of life expectancy in 2010-2015 and the lower number of observations in the data set for Cambodia compared to Switzerland, which makes the Cambodian curve more sensitive to outliers. The input data for Cambodia starts in 1950-1955, while the data for Switzerland starts in 1875-1880.

**Figure 4. 5 year gains in female life expectancy at birth (in years) by life expectancy at birth, simulation 4 and *2017 Revision*, Cambodia and Switzerland**

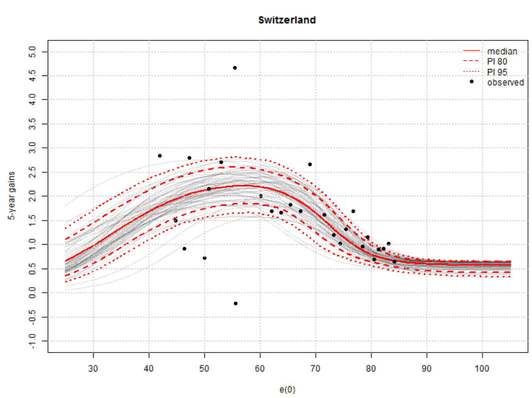
***Simulation 4*** ***2017 Revision***



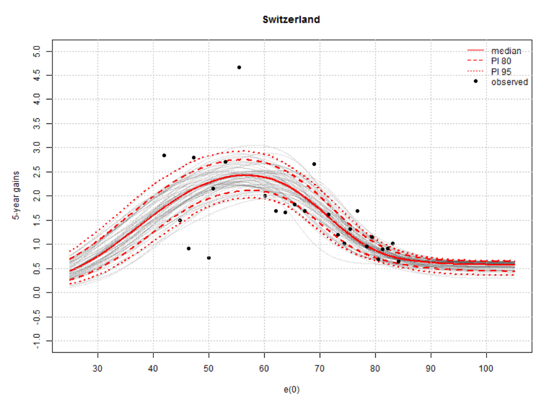

$\Delta_{c1}$= 17.53, $\Delta_{c2}$ = 36.38, $\Delta_{c3}$ = 10.58, $\Delta_{c4}$ = 16.56, $k^c$= 3.08, $z^c$ = 0.53




$\Delta_{c1}$= 24.75, $\Delta_{c2}$ = 33.96, $\Delta_{c3}$ = 5.02, $\Delta_{c4}$ = 18.60, $k^c$= 3.45, $z^c$ = 0.54




Δc1= 15.40, Δc2 = 35.91, Δc3 = 11.74, Δc4 = 18.54, kc= 2.41, zc = 0.57




Δ c1= 20.98, Δ c2 = 33.05, Δ c3 = 5.72, Δ c4 = 22.23, kc= 2.78, zc = 0.57

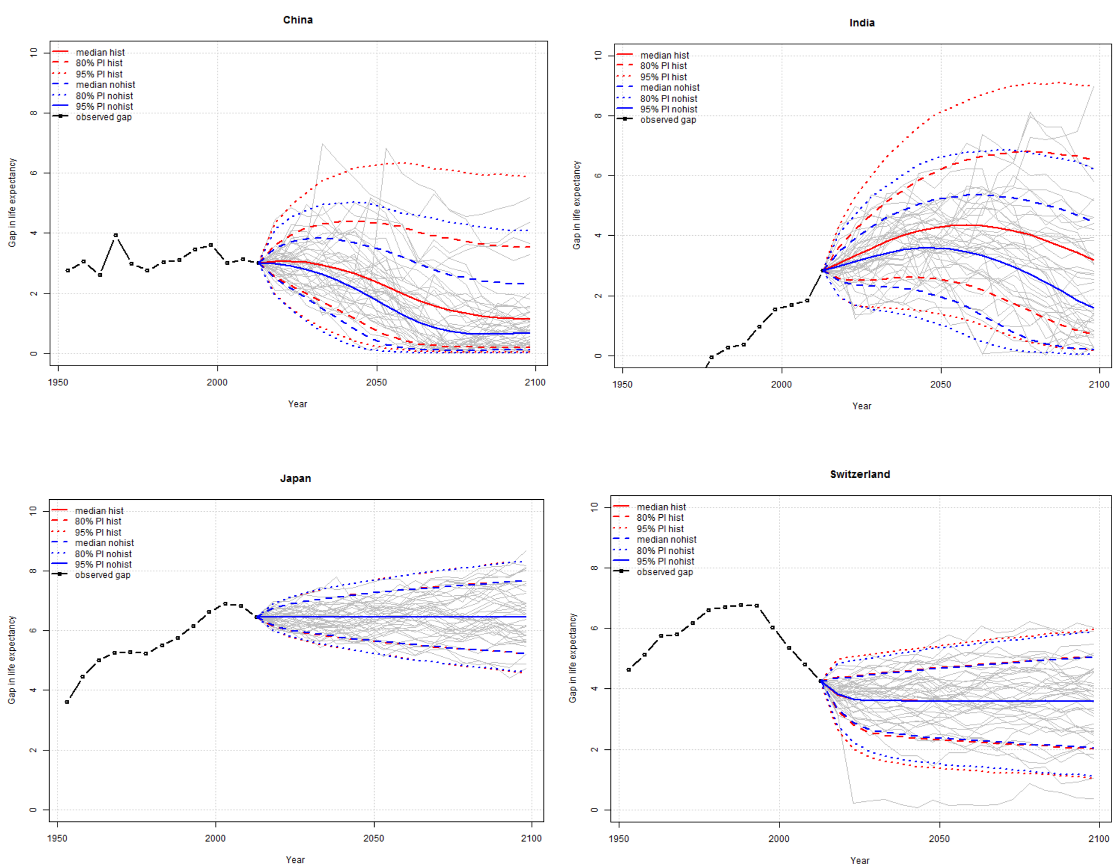
*Data Source:* United Nations, Department of Economic and Social Affairs, Population Division (2017c), *World Population Prospects: The 2017 Revision.*

NOTE: The observed five-year gains by level of life expectancy at birth (e(0)) are shown by black dots. For clarity, only 60 trajectories of the 1,100,000 calculated are shown here. The median projection is the solid red line, and the 80% and 95% prediction intervals are shown as dashed and dotted red lines respectively. In addition to estimates of female life expectancy at birth for the period 1950-2015 (based on the *2017 Revision*), historical data for pre-1950 periods are included. The estimated parameters underneath the figures represent the median value of the trajectories.

### *3.3. Changes in the sex gap model*

Figure 5 shows the effects of the updates in the sex gap model for selected countries, using historical data (prior to 1950). Overall, this modification contributed to a slower decline in the projected sex gap. As it can be observed in the cases of China and India, the median trajectory of the female-male gap in life expectancy at birth decreased more slowly in China or continued to increase for a longer period in India, when the historical data were included in the estimation of the parameters of the regression (red lines). Moreover, for these countries, the upper bounds of the projection interval increased considerably and the lower bounds were almost not affected when the historical data were used. Similar effects were observed for many countries after the inclusion of the historical data. For countries that are more advanced in the mortality transition, such as Japan or Switzerland, shown in the lower panels of figure 5, the median trajectory barely changed and the projection intervals (PIs) changed slightly in some cases.

**Figure 5. Female-male gap in life expectancy at birth (in years) and prediction intervals, with and without the inclusion of historical mortality data, China, India, Japan and Switzerland, 1950-2100**




*Data Source:* United Nations, Department of Economic and Social Affairs, Population Division (2017c), *World Population Prospects: The 2017 Revision.*

Overall, for countries with life expectancies close to 86 years in 2010-2015, the inclusion of the historical data in the estimation had almost no effect. This difference from countries with lower life expectancy is driven by the model's assumption of a constant gap when countries reach a female life expectancy of 86 years. Thus, at this level, the gap is no longer modelled by a linear regression, but it is kept constant with the standard deviation estimated using a normal distribution assumption.

The inclusion of the historical data in the *2017 Revision* provided greater consistency between the female life expectancy and the sex gap models. Additionally, in many countries it produced a slower decline of the female-male gap in life expectancy at birth in the projection period, producing a slightly wider gap between male and female life expectancies in some countries, or allowing for the gap to continue to increase for a longer period before declining. In the *2015 Revision*, the projected sex gap in life expectancy had reached relatively low levels in some countries, including in China, which contributed to atypical age-specific mortality patterns by sex.

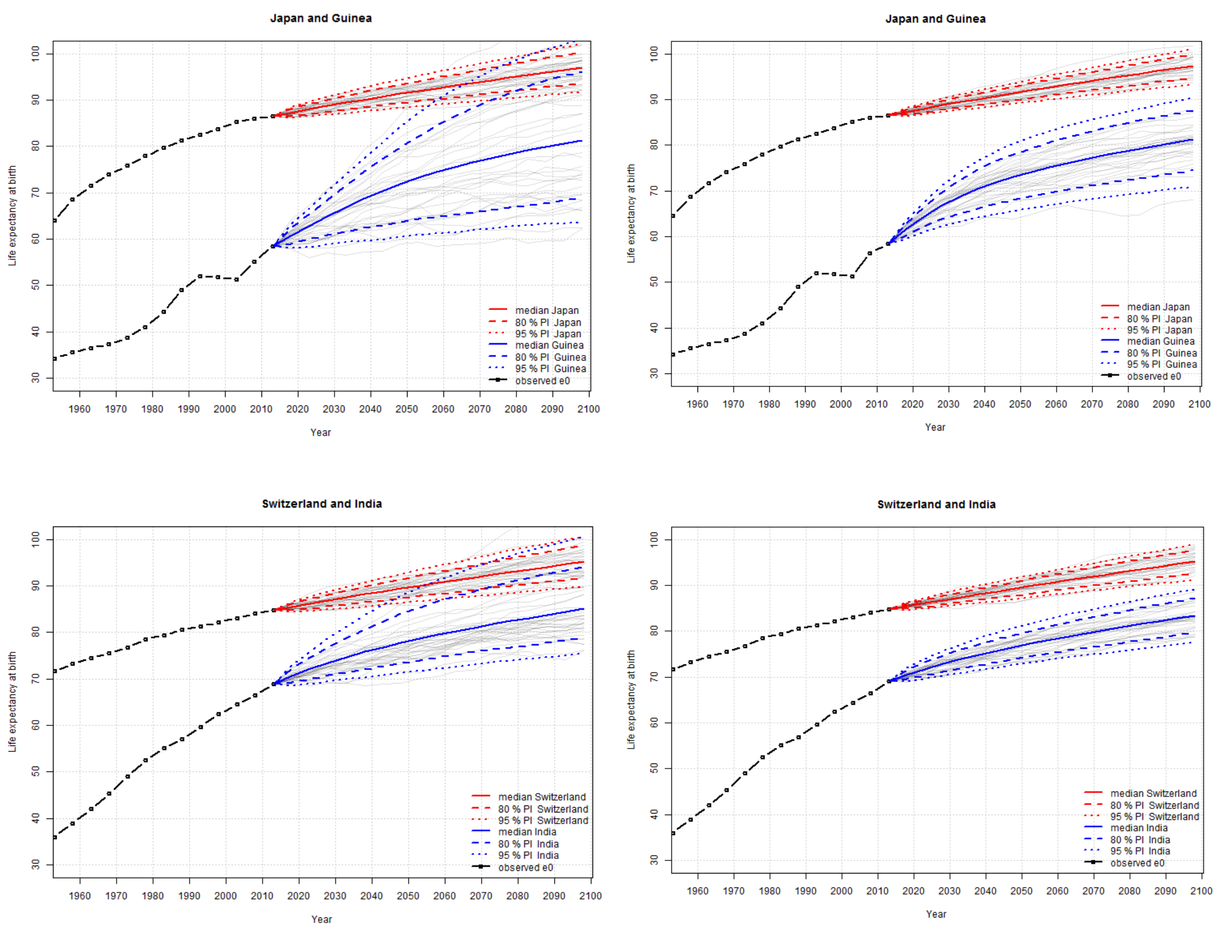
### *3.4. Comparing some results from the 2015 Revision and the 2017 Revision*

Changes made to the model in the *2017 Revision*, as described above, reduced considerably the RMSE and MAE, indicating a better fit. Moreover, the number of countries requiring adjustments in the distribution of their priors was reduced by more than 70 per cent, from 52 in the *2015 Revision* to 14 in the *2017 Revision*, indicating an improvement in the performance of the model. Countries are adjusted if their projected female life expectancy stood out either because of much faster or much slower improvements in life expectancy than typically experienced by other countries with similar levels of life expectancy in the 2010-2015 period (United Nations, 2017b). In order to illustrate the overall changes in the model, in this section a comparison of the posterior distributions of non-adjusted countries is provided for the *2015* and *2017 Revisions*. In general, the results of the *2017 Revision* are found to be more in line with what would be expected in future survival prospects based on the knowledge of current levels of life expectancy at birth and socio-economic and health conditions of individual countries.

Figure 6 compares the projected trajectories of female life expectancy at birth of the *2015* and *2017 Revisions* for Japan and Guinea, as well as for Switzerland and India. While the projected trajectories for Japan are quite similar across revisions, those for Guinea differ, especially for the upper 80 per cent and upper 95 per cent prediction intervals, which exceed Japan's lower prediction intervals. Although less extreme, the comparison for Switzerland and India also signals a similar pattern and an overall improvement in the trajectories across revisions. Such improvements in the trajectories have been identified in several countries.

**Figure 6. Female life expectancy at birth (in years) and prediction intervals, *2015* and *2017 Revisions,* Japan, Guinea, Switzerland and India, 1950 to 2100**

***2015 Revision*** ***2017 Revision***




*Data Sources:* United Nations, Department of Economic and Social Affairs, Population Division (2017c, d), *World Population Prospects: The 2017 Revision*; United Nations, Department of Economic and Social Affairs, Population Division (2015b, c), *World Population Prospects: The 2015 Revision.*

To further illustrate the intended improvement in the modelling process, table 2 includes the 30 countries or areas with the highest upper bounds of the 95 per cent prediction intervals of female life expectancy at birth in 2095-2100, for the *2015 Revision* and the *2017 Revision*. In the *2015 Revision*, all estimates were above 100 years while in the *2017 Revision*, they ranged from 97.0 years in New Zealand to above 100 years in Macao, Hong Kong, Japan and

Republic of Korea. Overall, the list from the *2017 Revision* is composed of countries that are also among today's world leaders in survival prospects. The list from the *2015 Revision*, in contrast, includes several countries (e.g. Senegal, Guinea, Mali, Somalia, etc.) that actually had quite low levels of life expectancy in 2010-2015.

TABLE 2. THIRTY COUNTRIES OR AREAS WITH THE HIGHEST UPPER BOUND OF 95 PER CENT PREDICTION INTERVALS OF FEMALE LIFE EXPECTANCY AT BIRTH IN 2095-2100, 2015 AND 2017 REVISION

*2015 Revision in 2095-2100**

| *Rank* | *Country or area* | *Life expectancy (years)* | *Rank* | *Country or area* | *Life expectancy (years)* |
|---|---|---|---|---|---|
| 1 | Singapore | 104.3 | 16 | Mongolia | 101.5 |
| 2 | Senegal | 103.6 | 17 | French Guiana | 101.5 |
| 3 | Martinique | 103.2 | 18 | France | 101.1 |
| 4 | Guinea | 103.2 | 19 | Luxembourg | 100.6 |
| 5 | Guam | 102.7 | 20 | Spain | 100.5 |
| 6 | Lebanon | 102.6 | 21 | Australia | 100.5 |
| 7 | Mali | 102.6 | 22 | Portugal | 100.5 |
| 8 | Guadeloupe | 102.3 | 23 | Cambodia | 100.5 |
| 9 | Chile | 102.0 | 24 | Switzerland | 100.4 |
| 10 | Japan | 102.0 | 25 | India | 100.4 |
| 11 | Israel | 101.9 | 26 | Haiti | 100.4 |
| 12 | Italy | 101.8 | 27 | Mayotte | 100.3 |
| 13 | China, Hong Kong SAR | 101.6 | 28 | Réunion | 100.2 |
| 14 | Western Sahara | 101.6 | 29 | Somalia | 100.2 |
| 15 | Republic of Korea | 101.6 | 30 | New Zealand | 100.1 |

*2017 Revision in 2095-2100*

| *Rank* | *Country or area* | *Life expectancy (years)* | *Rank* | *Country or area* | *Life expectancy (years)* |
|---|---|---|---|---|---|
| 1 | China, Macao SAR | 101.0 | 16 | Finland | 97.9 |
| 2 | China, Hong Kong SAR | 101.0 | 17 | Canada | 97.7 |
| 3 | Japan | 100.8 | 18 | Austria | 97.7 |
| 4 | Republic of Korea | 100.1 | 19 | Israel | 97.7 |
| 5 | Spain | 99.7 | 20 | Luxembourg | 97.7 |
| 6 | Martinique | 99.5 | 21 | French Guiana | 97.6 |
| 7 | Guadeloupe | 99.3 | 22 | Sweden | 97.5 |
| 8 | France | 99.1 | 23 | Slovenia | 97.4 |
| 9 | Singapore | 99.1 | 24 | Greece | 97.4 |
| 10 | Italy | 98.9 | 25 | Puerto Rico | 97.4 |
| 11 | Switzerland | 98.8 | 26 | Iceland | 97.4 |
| 12 | Australia | 98.4 | 27 | Norway | 97.3 |
| 13 | Portugal | 98.2 | 28 | Ireland | 97.2 |
| 14 | Mayotte | 98.1 | 29 | Belgium | 97.1 |
| 15 | Réunion | 98.0 | 30 | New Zealand | 97.0 |

*Data Sources:* United Nations, Department of Economic and Social Affairs, Population Division (2017d), *World Population Prospects: The 2017 Revision*; United Nations, Department of Economic and Social Affairs, Population Division (2015c), *World Population Prospects: The 2015 Revision.*

NOTES: Only countries or areas with 90,000 persons or more in 2015 or 2017 are considered.

*Excluding 6 "AIDS countries" with upper 95% PIs above 100 years. The ranking is based on the final estimates, that is taking into account adjustments that were made; without any adjustments of the prior distributions, the number of countries with life expectancy above 100 years would have been greater.

## 4. Discussion

During the *2017 Revision*, several aspects of the probabilistic models for the projections of life expectancy at birth and the sex gap in life expectancy were modified or updated, resulting in changes in projected life expectancy levels of countries, for both the median trajectories and associated uncertainty bounds (prediction intervals). The changes performed in the *2017 Revision* improved the model fit and the consistency of results throughout the projection, resulting in a reduction in the number of countries for which adjustments were made as compared to the *2015 Revision*.

Bayesian analysis requires providing *a priori* information of the parameters of the model. The different assumptions of the distribution of the priors used in the model for projecting female life expectancy, have an important role in the results, for both the median trajectory and the associated uncertainty bounds. Overall, they affect the countries differently depending on how advanced they are in the mortality transition, and the prior that is being changed. The modification in the assumption about the sum of deltas affects predominantly countries that have lower life expectancies at birth, and, overall, has greater implication on the trajectories in the upper bounds. The modification in the assumption about the $z$ factor affects predominantly countries that currently have higher levels of life expectancies at birth. In relation to the sex gap model, the use of historical data (prior to 1950) to estimate the parameters tends to modify the results, yielding a slower decline in the sex gap especially for countries that are at a less advanced stage in the mortality transition. The analysis that was conducted illustrates that the assumptions in the different modelling phases can influence the results of the projections.

When making projections of any indicator to a far horizon such as 2100, there is inherently a great deal of uncertainty. The paper presented an analysis on the extent to which the results are sensitive to the assumptions of the model and the extent that they can be modified to produce more consistent results. Measuring the uncertainty is not an easy task either and one could argue that there is some degree of uncertainty in the projected upper and lower bounds as well.

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